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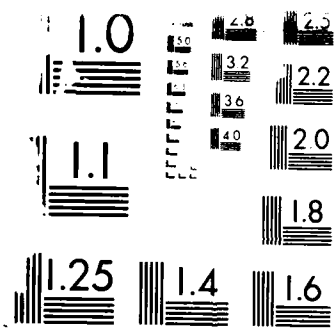
FAST ALGORITHMS FOR STRUCTURAL ANALYSIS LEAST SQUARES  
AND RELATED COMPUTATIONS(U) NORTH CAROLINA STATE UNIV  
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<p>OTIC FILE COPY</p> <p>This report summarizes the activities in support of the Air Force Project AFOSR-83-0255 during the past year. Efforts have been made to develop, test and analyze new fast techniques in numerical linear algebra for structural computations and least squares problems. Applications of this work include structural design and dynamics, least squares adjustments in geodesy, and least squares filtering in signal processing. Implementations and tests have been made on modern high performance architectures such as the Cray 2, Cray X-MP and Alliant FX/8 multiprocessors.</p>			
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I. RESEARCH OBJECTIVES

Considering the increasing demands on scientists and engineers to model, solve and analyze larger and more complex problems, the need for multiprocessing and/or vectorization of the numerical schemes being employed is substantial if significant time-to-solution reductions are to be achieved. It is not sufficient merely to implement old algorithms in a parallel processing environment. New fast algorithms for high speed computation on the modern generation of supercomputers is essential. In order to meet these challenges the research objectives of this AFOSR project are to develop new techniques in numerical linear algebra and its applications for implementation on these new architectures. Significantly, applications of our work to practical problems of structural analysis and design and to least squares adjustments, estimation and digital filtering are also being investigated.

Our current objectives in structural analysis are to develop efficient and stable high speed algorithms for the design and analysis of large complex systems. Our interest here is in developing stable alternatives to the often ill-conditioned stiffness matrix approach to solving problems in elastic analysis and structural dynamics. For example, the principal investigator and Michael Berry at the University of Illinois Center for Supercomputer Research and Development are developing a comparative study of the performances of seven alternative methods to the stiffness approach on the Alliant FX/8 and Cray X-MP systems. These methods involve various orthogonal factorization approaches as well as preconditioned conjugate gradient methods which completely avoid formation of the stiffness equations. The results of this study will be presented this Fall as an invited paper at the World Congress on Computational Mechanics.

Our work involving least squares problems has several objectives. We wish to implement and test a recent parallel block scheme by Golub, Sameh

and the principal investigator, on the Alliant FX/8 multiprocessor. We have in mind here large scale geodetic adjustment computations. We are also implementing and testing new conjugate gradient type algorithms by Barlow, Nichols and the principal investigator on the Alliant. A new quadratically convergent parallel algorithm based upon Newton's method, by Wright and the principal investigator, has just been implemented on the Alliant. Our final least squares objective during this period has been to complete the error analysis and testing of a recursive least squares hyperbolic rotation algorithm for signal processing. This is joint work with a Ph.D. student, C. Pan, and with T. Alexander from the NCSU Department of Electrical and Computer Engineering. Our scheme is amenable to implementation on a variety of vector and parallel processing systems, such as the Alliant.

Some major results obtained during the past year of this project are outlined in the next section.

## II. SUMMARY OF MAJOR RESULTS

Our most important research accomplishments during the past year are briefly described below. These results have been obtained on four general problems in numerical linear algebra and its applications. Preprints detailing this work have been provided to the AFOSR.

### 1. Parallel Multisplitting Iterative Methods (Joint with M. Neumann)

Despite the major activity recently on parallel processing, relatively few effective new algorithms designed exclusively for multiprocessors have been put forth. One such new algorithm is the multisplitting iterative algorithm suggested by O'Leary and White. Although O'Leary and White, and later White, have given some sufficient conditions for convergence, a general convergence theory has not been developed even for the classical situation where the coefficient matrix  $A$  is an  $M$ -matrix or is symmetric positive definite.

Our purpose in this paper is to study the M-matrix case in detail. The multisplitting process for  $A \in \mathbb{R}^{n,n}$  is recast as an ordinary iterative process for a certain block matrix  $A \in \mathbb{R}^{kn, kn}$ , where  $k$  is the number of processors, and standard convergence results are used to develop a convergence theory for multisplitting iterative methods where  $A$  is an M-matrix.

Comparison results between multisplitting methods are established in terms of monotonic norms and, for the case where  $A$  is irreducible, in terms of the asymptotic convergence rate. A key observation here is that in certain cases the rate of global convergence of these parallel iterative methods is inherent in the splitting of  $A$  and is independent of the manner in which the work is distributed among the processors. Thus in general one can distribute the work for load balancing purposes without affecting the convergence rate.

2. Conjugate Gradient Method for Equality Constrained Least Squares with Applications to Structural Analysis (Joint with J. Barlow and N. Nichols)

A preconditioned conjugate gradient algorithm has been developed for solving large scale least squares problems with equality constraints. The method has been implemented, tested and compared with other methods on the Alliant FX/8 and Cray X-MP systems for solving large scale problems in structural optimization and design. This method can be applied to both full rank and rank deficient applications in structural analysis. Comparisons with various other approaches including a recent weighting method by Van Loan are made on a testbed of structural analysis data.

3. Parallel Block Schemes for Large Scale Least Squares Computations (Joint with G. Golub and A. Sameh)

Large scale least squares computations arise in a variety of scientific and engineering problems, including geodetic adjustments and surveys,

medical image analysis, molecular structures, partial differential equations and substructuring methods in structural engineering. In each of these problems, matrices often arise which possess a block structure which reflects the local connection nature of the underlying physical problem. For example, super-large nonlinear least squares computations currently arise in geodesy. Here the coordinates of positions are calculated by iteratively solving overdetermined systems of nonlinear equations by the Gauss-Newton method. The U.S. National Geodetic Survey will complete this year (1986) the readjustment of the North American Datum, a problem which involves over 540 thousand unknowns and over 6.5 million observations (equations). The observation matrix for the least squares computations has a block angular form with 161 diagonal blocks, each containing 3 to 4 thousand unknowns. In this paper parallel schemes are suggested for the orthogonal factorization of matrices in block angular form and for the associated backsubstitution phase of the least squares computations. In addition, a parallel scheme for the calculation of certain elements of the covariance matrix for such problems is described. It is shown that these algorithms are ideally suited for multiprocessors with three levels of parallelism such as the Cedar system at the University of Illinois.

4. Analysis and Testing of Fast Recursive Least Squares Filtering Algorithm in Signal Processing (Joint with T. Alexander and C. Pan)

The application of hyperbolic plane rotations to the least squares down-dating problem arising in windowed recursive least squares signal processing is studied. A backward error analysis under some simplifying assumptions is used to show that this method can be expected to perform well in the presence of rounding errors, provided that the problem is not too ill-conditioned. It is shown in detail how the method's stability depends upon the conditioning. The results here contrast with the recent error analyses of downdating methods by Bojanczyk, Brent, Van dooren

and de Hoog, who suggest mixed rather than backward stability bounds. Comparisons are made with the usual method based upon orthogonal rotations as implemented in LINPACK. Both methods have the important advantage over the classical normal equations approach in that they can be effectively implemented on special purpose signal processing devices requiring shorter wordlengths. However, the hyperbolic rotation method requires  $n^2$  fewer multiplications and additions for each downdating step than the orthogonal rotation method, where  $n$  is the number of least squares filter coefficients. In addition, it is more amenable to implementation on a variety of vector and parallel machines. In many signal processing applications  $n$  is not large and if  $n$  processors are available, then the downdating process can be accomplished in  $2n$  time steps by the hyperbolic rotation methods.

### III. RESEARCH IN PROGRESS

Our research projects in support of this grant which are currently underway are briefly described below. Preprints of research papers providing complete description of the results of these projects will soon be available.

1. A Robust Parallel Algorithm for Minimizing a Weighted Sum of Euclidean Norms (Joint with S. Wright)

A robust, quadratically convergent parallel algorithm is being developed for solving the nonlinear problem

$$\min_x \sum_{i=1}^s ||b_i - A_i x||_2$$

where the  $A_i$  are  $m_i \times n$  matrices with full column rank  $n$ ,  $1 \leq i \leq s$ .

Applications arise in facility location problems, in geodetic adjustments and in surface fitting problems. The algorithm has been implemented and testing is underway on an Alliant FX/8 vector multiprocessor system.



2. Preconditioned Conjugate Gradients by Incomplete Hyperbolic Reduction  
(Joint with D. Pierce)

A new conjugate gradient algorithm based in part upon SSOR preconditioning is being investigated. The novel feature of our approach is the use of stable hyperbolic rotation incomplete factorization techniques to enhance the convergence properties. The method is being implemented on an Alliant FX/8 and tested using a testbed of structural analysis data.

3. Geodetic Least Squares Adjustment Techniques on the Cedar System  
(Joint with W. Harrod and A. Sameh)

Our purpose is to implement and test a parallel block orthogonal factorization scheme on the Cedar multiprocessor system being developed at the University of Illinois Center for Supercomputer Research and Development. The first phase of this project includes implementation on the Alliant FX/8 system which will form the "clusters" for the Cedar machine. Tests will be made using geodetic data supplied by the National Geodetic Survey and by the Defense Mapping Agency.

4. Parallel Algorithms and Experiments for Structural Analysis (Joint with M. Berry)

The implementation of direct and iterative methods for the solution of elastic analysis problems on the Alliant FX/8 and the CRAY X-MP/24, is underway. The direct methods include the classical displacement method, the natural factor method by Argyris, and weighted least squares methods by Van Loan. The iterative methods include a preconditioned conjugate gradient method for constrained least squares problems by Barlow/Nichols/Plemmons and preconditioned conjugate gradient method for weighted least squares equations by Freund.

Performance tests on the Alliant FX/8 are being conducted to determine which method(s) is(are) optimal for parallelization and speedup. Comparing the accuracy of the force vectors and the timings of the iterative schemes with the direct schemes on two-dimensional frame problems, we can expect lower execution times for the iterative methods if we accept force vectors that yield high precision residuals with lower precision quadratic forms.

While the performance of the two conjugate gradient schemes are approximately the same, the Barlow/Nichols/Plemmons scheme is more advantageous in the fact that no weighting of the equilibrium matrix is required. Some of the parallel algorithms that are being experimentally used in all these methods include block Cholesky factorization, block Householder QR factorization, and pipelined-Givens reduction. Comparisons in speed with the appropriate routines from LINPACK are also being made. The vectorization potential of the methods is being determined by the implementations on the CRAY X-MP/24. All results thus far are preliminary and further code revisions are necessary for both the Alliant FX/8 and the CRAY X-MP/24. The results of this short-term project will be described in a paper which will be presented at the First World Congress on Computational Mechanics in Austin, Texas in September.

#### IV. TECHNICAL PUBLICATIONS

1. "A parallel block iterative scheme applied to computations in structural analysis", SIAM J. Alg. and Disc. Meth., 7(1986), 337-347.
2. "Convergent iterations for computing stationary distributions of Markov chains", SIAM J. Alg. and Disc. Meth., 7(1986), 390-398 (with G. Barker).

3. "An algorithm to compute a sparse basis of the null space", Numerische Math. 47(1985), 483-504 (with M. Berry, M. Heath, I. Kaneko, M. Lawo and R. Ward).
4. "Updating LU factorizations for computing stationary distributions", SIAM J. Alg. Disc. Meth., 7(1986), 30-42 (with R. Funderlic).
5. "Convergence of parallel multisplitting iterative methods for M-matrices", to appear in Lin. Alg. and Applic. (with M. Neumann).
6. "A conjugate gradient method for the solution of equality constrained least squares problems", to appear in the Proc. of the 1986 SPIE Conf., San Diego, CA (with J. Barlow and N. Nichols).
7. "Parallel block schemes for large scale least squares computations", to appear in the Proc. of the Workshop on Scientific Applic. and Alg. Design for High Speed Computing, Urbana, IL (with G. Golub and A. Sameh).
8. "Analysis of a recursive least squares hyperbolic rotation algorithm for signal processing", submitted to the Lin. Alg. and Applic. Special Issue on Applic. to Elec. Eng. (with S. Alexander and C. Pan).

#### V. PERSONNEL ASSOCIATED WITH THE RESEARCH EFFORT

- R. J. Plemmons, Principal Investigator (1 mo. summer, 1 mo. academic year)  
R. B. Mattingly, GRA (1/2 time)  
D. J. Pierce, GRA (1/2 time), Ph.D. expected Fall 1986 or Spring 1987.

#### VI. CONFERENCE AND COLLOQUIUM ACTIVITIES

1. Colloquium Lecture - "Parallel algorithms in structural analysis", University of Virginia, October 1985.
2. Contributed Lecture - "Parallel algorithms for least squares problems in dual angular form", SIAM Conference on Parallel Processing, Norfolk, Virginia, November 1985 (Presented by D. Pierce).

3. Contributed Lecture - "Some parallel algorithms for matrix structural analysis", SIAM Conference on Parallel Processing, Norfolk, Virginia, November 1985.
4. Invited Lecture - "Multisplitting parallel iterative methods", NSF Conference on Matrix Theory, Auburn, AL, March 1986.
5. Invited Panelist - "Geodetic computations", Workshop on Scientific Applications and Algorithm Design for High Speed Computation, Urbana, IL, April 1986.
6. Colloquium Lecture - "Parallel iterative methods", Wake Forest University, April 1986.
7. Invited Lecture - "Analysis of a fast recursive least squares filtering algorithm in signal processing", Workshop on Communications and Signal Processing, Raleigh, NC, May 1986.
8. Conference Organizing Committees -
  - (a) SIAM Conference on Linear Algebra in Signals, Systems and Control, Boston, MA, August 1986.
  - (b) Invited Special Session - "Advances in Parallel Processing", World Congress on Computational Mechanics, Austin, TX, September 1986.
  - (c) Third SIAM Conference on Applied Linear Algebra, Madison, WI, May 1987.

## VII. SUMMARY

To summarize, the activities described in this Interim Annual Report represent our efforts to develop, analyze and test fast algorithms for structural analysis and least squares problems. Special features of the problems are being addressed and implementations are being made on modern high performance architectures such as the CRAY 2, CRAY X-MP and Alliant FX/8 multiprocessors.

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